TSUNAMI WAVE ENERGY

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RÉSUMÉ
Dans l'abondante littérature consacrée à la recherche sur les tsunamis, on trouve relativement peu d'articles consacrés aux questions d'énergie. Nous procédons à une étude théorique de l'énergie des vagues déclenchées par mouvement du fond en partant des équations complètes d'Euler avec surface libre puis nous comparons les résultats obtenus par différents modèles d'approximation : équations de Saint-Venant, systèmes de Boussinesq. Les effets dispersifs n'apparaissent qu'à un ordre supérieur dans le bilan d'énergie.

ABSTRACT
In the vast literature on tsunami research, relatively few articles have been devoted to energy issues. A theoretical investigation on the energy of waves generated by bottom motion is performed here. We start with the full Euler equations in the presence of a free surface. Then we compare the results obtained through various approximations : nonlinear shallow water equations, Boussinesq systems. It is shown that dispersive effects only appear at higher order in the energy budget.

1. INTRODUCTION

Oceanic waves can be devastating as shown by recent events. Whilst some areas are more vulnerable than others, the recent history shows that catastrophic waves can hit even where they are not expected. The tsunami waves generated by the huge undersea earthquake in Indonesia on 26 December 2004 caused devastation across most of the coasts of the Bay of Bengal. The tsunami waves generated by the massive submarine landslide in Papua-New Guinea on 17 July 1998 as well as the 17 July 2006 Java tsunami and the 2 April 2007 Solomon Islands tsunami also caused devastation, but on a smaller scale. Unfortunately, such cataclysmic tsunamis are likely to be generated again by earthquakes, massive landslides or volcano eruptions (see for example Synolakis & Bernard 2006 for an excellent review).

Information on tsunami energy can be obtained by applying the normal mode representation of tsunami waves, as introduced by Ward (1980). For example, Okal (2003) considers the total energy released into tsunami waves. He obtains expressions for the energy of tsunamis (see the expressions (31) and (36) in his paper). In the case of a landslide, he computes the ratio between tsunami energy and total change in energy due to the slide. In the present paper, we will use the incompressible fluid dynamics equations. Tsunamis have traditionally been considered as non-dispersive long waves. However various types of data (satellite data – see for example Kulikov, Medvedev & Lappo 2005, hydrophone records – see for example Okal, Talandier & Reymond 2007) based on the 2004 Sumatra tsunami indicate that tsunamis are made up of a very long dispersive wave train, especially when they have enough time to propagate. These waves travel across the ocean surface in all directions away from the generation region. Recent numerical computations using dispersive wave models such as the Boussinesq equations show as much as 20% reduction of tsunami amplitude in certain locations due to dispersion (see for example Dao & Tkalich 2007). But one has to be careful with the interpretation of satellite data: as indicated by Kânoglu & Synolakis (2006), the mid-ocean steepness of the 2004 Sumatra tsunami measured from satellite altimeter data was less than 10^{-5}. Nonlinear dispersive theory is necessary only when examining steep gravity waves, which is not the case in deep water.

The wavelength of tsunamis and, consequently, their period depend essentially on the source mechanism. If the tsunami is generated by a large and shallow earthquake, its initial wavelength and period will be greater. On the other hand, if the tsunami is caused by a landslide (which happens less commonly but can be devastating as well), both its initial wavelength and period will be shorter. From these empirical considerations one can conclude that dispersive effects are a priori more important for submarine landslide and slump scenarios than for tsunamigenic earthquakes.

Once a tsunami has been generated, its energy is distributed throughout the water column. Due to the large scale of this amazing natural phenomena and limited power of computers, tsunami wave modellers have to adopt some kind of simplified models which reduce a fully three-dimensional (3D) problem to a two-dimensional (2D) one. This approach is natural, since in the case of very long waves the water column moves as a whole. Consequently the flow is almost 2D. Among these models one can mention the nonlinear shallow water equations, Boussinesq type models, Green-Naghdi equations and Serre equations. Note that there is a wide variety of models, depending on whether or not they include run-up/run-down, bottom friction, turbulence, Coriolis effects, tidal effects, etc.

At present time scientists can easily predict when a tsunami will arrive at various places by knowing rough source characteristics and bathymetry data along the paths to those

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places. Unfortunately one does not know as much about the energy propagation of such waves. Obviously tsunami amplitude is enhanced over the major oceanic ridges. As emphasized by Kowalik *et al.* (2007), travel-time computation based on the first arrival time may lead to errors in the prediction of tsunami arrival time as higher energy waves propagate slower along ridges. How is energy distributed during the first seconds of a tsunami? The purpose of this study is to shed some light on energy propagation and to see if the importance of dispersion in tsunamis can be studied by looking at the energy rather than at wave profiles.

Previous researchers have considered this topic. Recently there was an attempt to obtain equations for tsunami energy propagation. We can mention here the work of Tinti and Bortolucci (2000) devoted to idealized theoretical cases and the work of Kowalik *et al.* (2007) using the energy flux point of view to study the changes in the 2004 Sumatra tsunami signal as it travelled from Indonesia to the Pacific Ocean. We believe that these models can be improved, given the present state of the art in wave modelling.

A point of interest is that some of the equations used for wave modelling have an infinite number of conserved quantities. There has been some confusion in the literature on which quantities can be called energy. Indeed there is here an interesting question. In incompressible fluid mechanics, the internal energy equation is decoupled from the equation of continuity and from the fundamental law of dynamics. It is used only when one is interested in computing the temperature field once the velocity distribution is known. In addition to the internal energy equation, one can write a total energy (internal energy + kinetic energy) equation, or a total enthalpy equation. The confusing part is that for perfect fluids one usually defines the total energy differently: it is the sum of internal energy, kinetic energy, and potential energies associated to body forces such as gravitational forces and to the pressure field. If in addition the fluid is incompressible, then the internal energy remains constant. In the classical textbooks on water waves (Stoker 1958, Johnson 1997), one usually introduces the energy E as the sum of kinetic energy and potential energy and then looks for a partial differential equation giving dE/dt (incidentally the meaning of d/dt is not always clearly defined). In any case, when one uses a depth-integrated model such as the nonlinear shallow water equations, one can compute the energy a posteriori (the potential energy is based on the free-surface elevation and the kinetic energy on the horizontal velocity). But one could also apply the nonlinear shallow water assumptions on the full energy equation to start with. Then one would obtain a nonlinear shallow water approximation of the energy equations. Are these two approaches equivalent?

First we present the energy equation. Then we see what happens to the energy equation under the nonlinear shallow water assumptions. Finally we present some numerical computations.

2. ENERGY EQUATION

The fluid is assumed to be inviscid. Its motion is governed by the three-dimensional (3D) Euler equations, written here in their incompressible form:

$$\begin{aligned} &\nabla \cdot u = 0 \\ &\frac{\partial u}{\partial t} + \nabla \cdot \left(u \otimes u + \frac{p}{\rho} Id \right) = g \\ &\frac{\partial E}{\partial t} + \nabla \cdot \left[\left(E + gz + \frac{p}{\rho} \right) u \right] = 0 \end{aligned} \quad (1)$$

where ρ is the fluid density, u the velocity vector, *E* the total energy e + ½ u^2, p the pressure and g the acceleration due to gravity. The third equation is in fact redundant in incompressible fluid mechanics. However we keep it since we are going to work directly on it. The fluid domain is bounded above by the free surface z = η(x,y,t) and below by the moving bottom z = – h(x,y,t). Below we denote the horizontal gradient by $\nabla_\perp$ and the horizontal velocity by $u_\perp$. After a few manipulations and integration across the water column from bottom to top, one can write the following energy equation:

$$\mathrm{E}_t + \nabla_\perp \cdot \Phi + \mathrm{P} = 0 \quad (2)$$

where E is the energy (sum of kinetic and potential energy) in the flow (not to be confused with *E*, the total energy), per unit horizontal area, Φ the horizontal energy flux vector, and P the net energy input due to the pressure forces doing work on the upper and lower boundaries of the fluid. They are given by the following expressions:

$$\begin{aligned} &\mathrm{E} = \int_{-h}^{\eta} \left(\frac{1}{2} \rho u \cdot u + \rho g z \right) dz \\ &\Phi = \int_{-h}^{\eta} u_\perp \left(\frac{1}{2} \rho u \cdot u + p + \rho g z \right) dz \\ &\mathrm{P} = p_s \eta_t + p_b h_t \end{aligned} \quad (3)$$

where p_s is the pressure exerted on the free surface (due to wind for example) and p_b the bottom pressure. In the case of a stationary bottom boundary and of a free surface on which the pressure vanishes, then as expected the net energy input P is identically zero. Energy can be brought to the system by a moving bottom or by a pressure disturbance on the free surface.

In the next section, we perform the classical shallow water approximation and we see what happens to the energy equation (2).

3. SHALLOW WATER APPROXIMATION

The problem of tsunami propagation possesses two characteristic length scales: the average water depth h_0 for the vertical dimension and a typical wavelength L for the horizontal dimensions. These two lengths are used to introduce dimensionless independent variables. In order to introduce the dimensionless dependent variables we need one more length, namely the typical wave amplitude a. The shallow water approximation is then based on the introduction of two small parameters: $\varepsilon = a/h_0$ and $\mu = (h_0/L)^2$. The parameter ε represents the relative importance of nonlinear terms and μ measures the relative importance of dispersive effects. Following Ursell, we also introduce a number which measures the relative importance of nonlinear and dispersive effects in long waves: $S = \varepsilon/\mu$. Then one can write the equations in dimensionless form. The energy equation becomes

$$\mathrm{E}_t + \varepsilon\nabla_\perp \cdot \Phi + \mathrm{P} = 0 \qquad (4)$$

If the bottom moves according to

$$h(x,y,t) = h_0(x,y) + \varepsilon\xi(x,y,t) \qquad (5)$$

then the energy equation becomes

$$\mathrm{E}_t + \varepsilon\nabla_\perp \cdot \Phi + \varepsilon\eta\xi_t = 0 \qquad (6)$$

Note that we have simplified the expression for the pressure by keeping only its leading order term and assuming that ε and μ are of the same order. Terms involving μ show up only at next order, so one can conclude that dispersion comes as a second-order effect in the energy balance.

4. NUMERICAL COMPUTATIONS

We integrate numerically the nonlinear shallow water equations, including the energy equation that we derived above. The system looks like

$$\begin{aligned} &\eta_t + \nabla_\perp \cdot ((h+\varepsilon\eta)u_\perp) = -\xi_t \\ &u_{\perp t} + \frac{1}{2}\varepsilon\nabla u_\perp^2 + \nabla\eta = 0 \\ &\mathrm{E}_t + \varepsilon\nabla_\perp \cdot \Phi + \varepsilon\eta\xi_t = 0 \end{aligned} \qquad (7)$$

The finite volume method is used to integrate the system numerically. For the numerical flux, we use the characteristic flux, introduced by Ghidaglia, Kumbaro & Le Coq (1996) – see also Ghidaglia *et al.* (2001). This scheme is easy to implement, it is not based on an exact solution to the Riemann problem and it can be extended naturally to multi-dimensional problems. We generate the waves by moving the sea bottom as it may occur in reality. The displacements are constructed in the following way. The main ingredient is Okada's solution (Okada 1985). The moving bathymetry is obtained as follows:

$$h(x,y,t) = h_0(x,y) - (1 - e^{-\alpha t})\xi(x,y) \qquad (8)$$

where $\xi(x,y)$ is the seabed static deformation prescribed by Okada's solution. The parameter α is related to the characteristic time of the deformation. For the numerical application, the fault depth is 3 km, the dip angle is 13°, the fault length is 6 km, the fault width 4 km, the magnitude of Burger's vector is 10 m, the water depth 4 km. The figures show the free-surface elevation together with the depth-averaged total energy at various times.

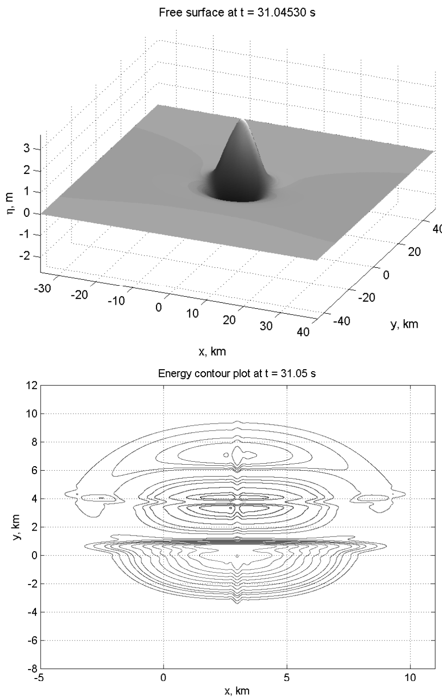

Figure 1. Free-surface snapshot and energy contour plot around the tsunami generation region. At t=31s.

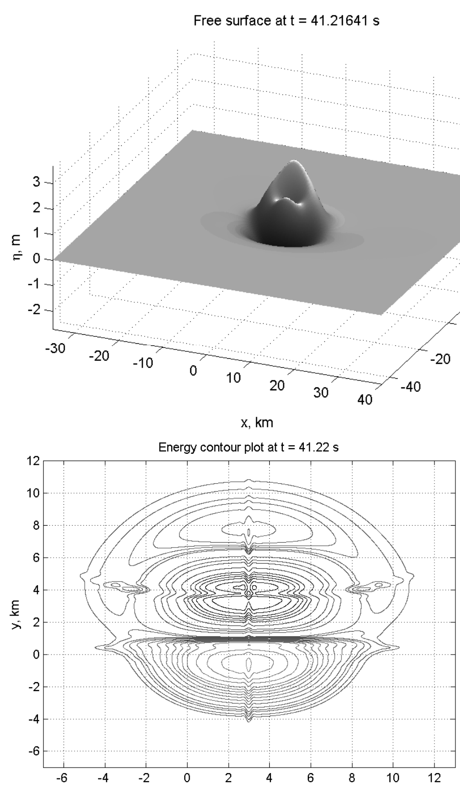

Figure 2. Same as figure 1 at t=41s.

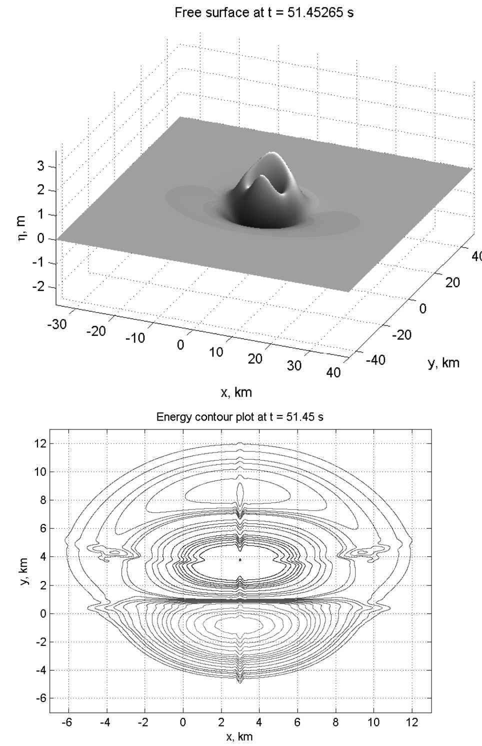

Figure 3. Same as figure 1 at t=51s.

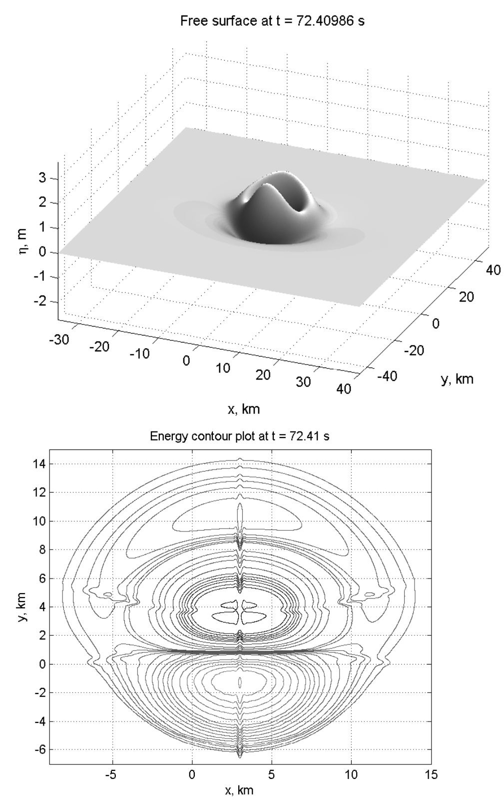

Figure 4. Same as figure 1 at t=72s.

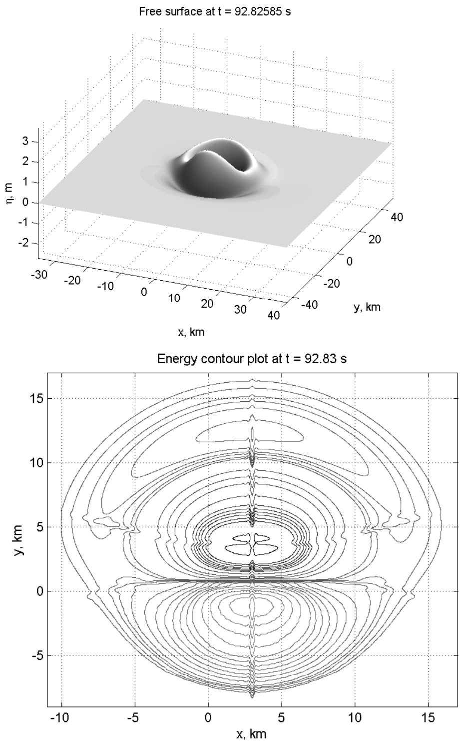

Figure 5. Same as figure 1 at t=92s.

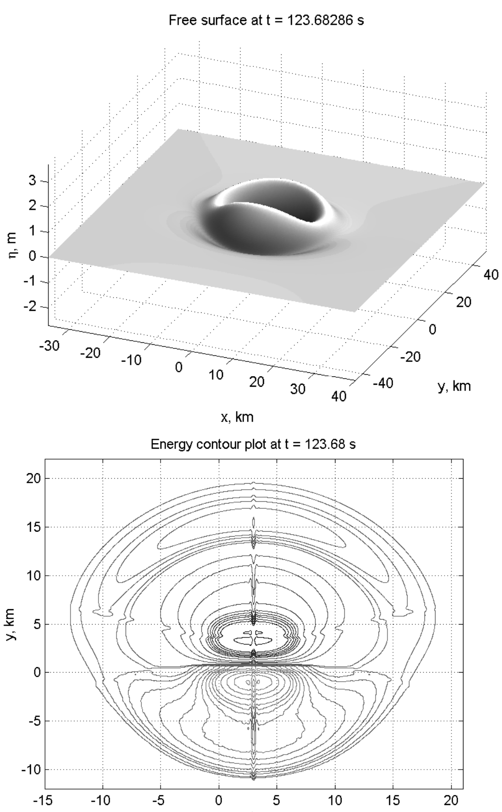

Figure 6. Same as figure 1 at t=123s.

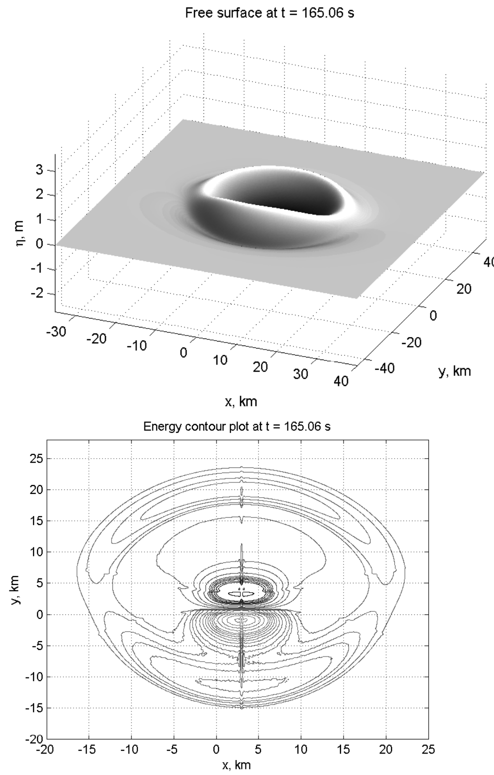

Figure 7. Same as figure 1 at t=165s.

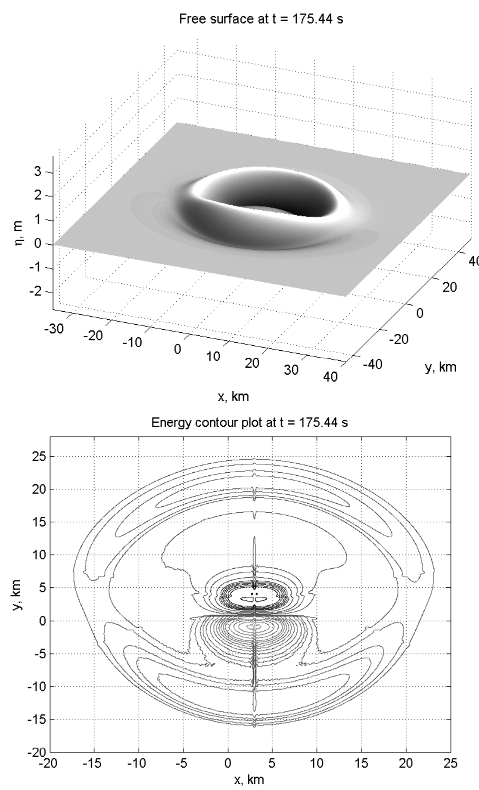

Figure 8. Same as figure 1 at t=175s.

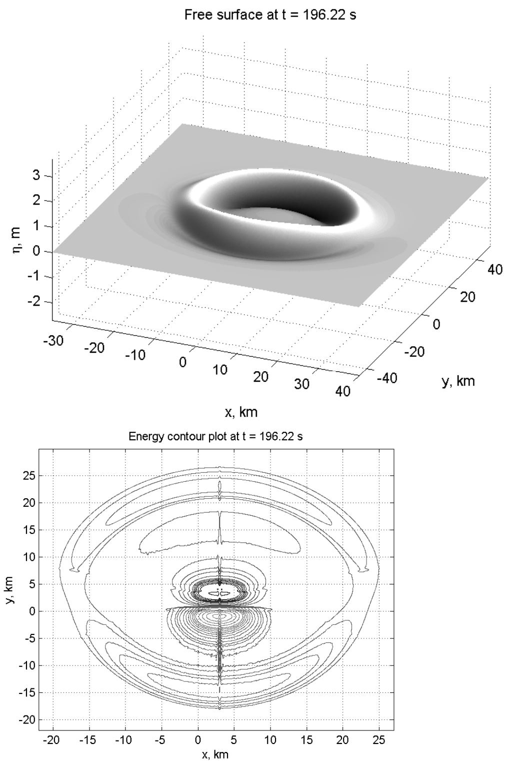

Figure 9. Same as figure 1 at t=196s.

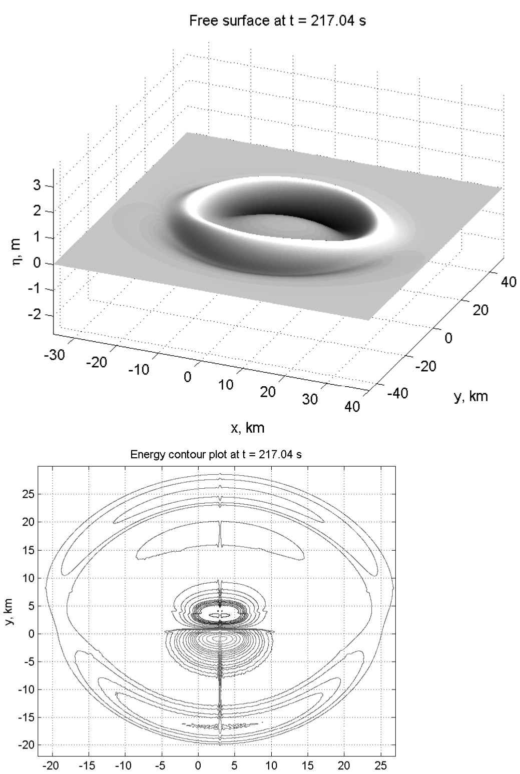

Figure 10. Same as figure 1 at t=217s.

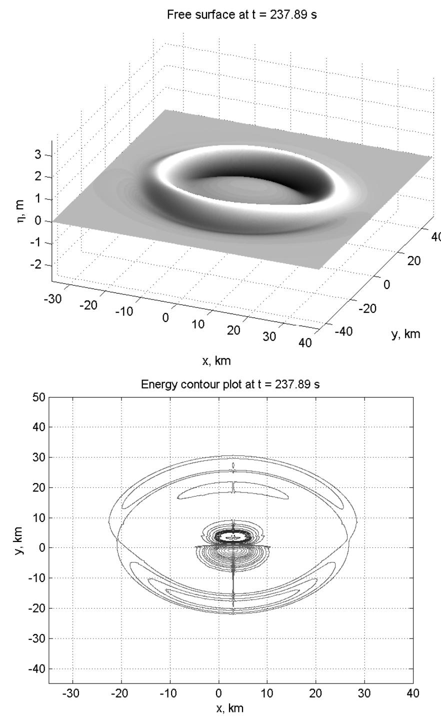

Figure 11. Same as figure 1 at t=237s.

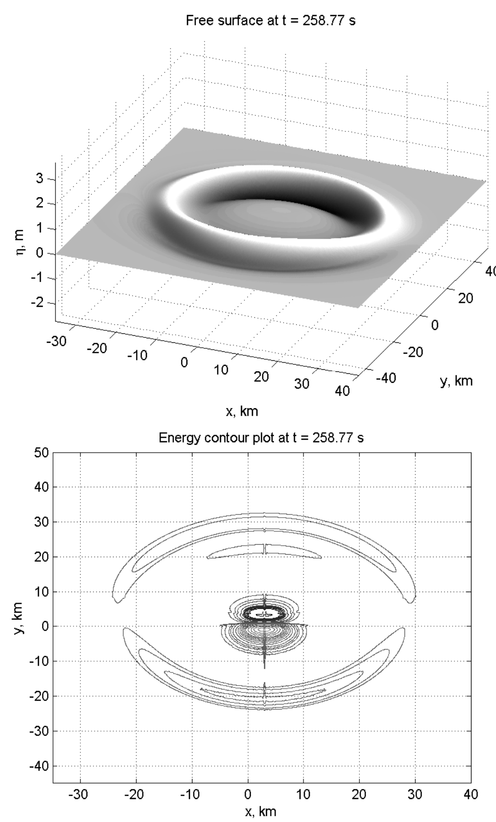

Figure 12. Same as figure 1 at t=258s.

5. CONCLUSIONS

In this short paper we provide a rigorous treatment of the energy equation for tsunami propagation. The resulting equation does not coincide exactly with previous equations that have been used to study the propagation of tsunami energy. Indeed it is not equivalent to try to obtain the energy equation from the shallow-water equations. What we do is to apply the shallow water approximation directly to the energy equation.

The present paper should be considered only as a preliminary work on the topic of tsunami wave energy. Deeper mathematical and especially physical analysis is needed. For example, it seems that it is not the whole energy which propagates with the wave. There is a part that remains trapped in the generation region, as indicated by our numerical computations.

We attempted to understand the energy transfer from the moving bottom to the water. The importance of this topic can be justified by the hazard that tsunamis represent for coastal regions.

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