

NUMERICAL MODELING OF UNDERWATER ROCKFALL

Dominique Turmel, Jacques Locat

Laboratoire d'études sur les risques naturels (LERN), Département de géologie et de génie géologique, Université Laval, Québec, Canada, G1K 7P4, dominique.turmel.1@ulaval.ca

RÉSUMÉ

Tout comme le milieu aérien, le milieu marin peut présenter des environnements où des talus rocheux sont formés suite à des processus gravitaires impliquant des chutes de blocs. C'est le cas, par exemple, le long de fjords ou de canyons sous-marins. Dans l'analyse de ce processus gravitaire en milieu sous-marin, l'effet du fluide ambiant doit être considéré. Un modèle numérique a été développé afin de modéliser le comportement en deux dimensions des chutes de blocs dans un liquide Newtonien, tel que l'eau ou l'air. L'écoulement incompressible et turbulent du fluide autour du bloc est résolu avec une méthode itérative couplant la pression et la vitesse, sur un maillage dynamique non-structuré en trois dimensions composé de polyèdres. Les forces dues à la pression, au cisaillement, à la turbulence ainsi que la force de gravité sont calculées autour du bloc et ensuite appliquées à ce bloc afin de définir son mouvement. Dans cet article, le comportement des chutes de blocs dans l'eau sera comparé à celui en milieu aérien dans trois situations différentes : la chute libre, le rebond sur une surface horizontale ainsi que le rebond sur une surface inclinée.

ABSTRACT

The marine environment presents various settings in which talus slopes are formed via gravity driven rockfall processes similar to what exists on land. This is the case along fjords or submarine canyons, where submarine installations or natural sanctuaries, such as coral reef, need to be protected. In a submarine rockfall analysis, the effect of the ambient fluid, in addition to hydrodynamics constraints and water currents, must be considered. A finite volume model has been developed to model 2D rockfall behaviour in a Newtonian fluid, such as water or air. Turbulent and incompressible flow around the moving rock is solved with a pressure-velocity iterative method on a dynamic 3D unstructured mesh of polyhedral cells. Pressure force, shear force, turbulent forces and gravity force are then applied to the moving rock. In this paper we will compare underwater rockfall behaviour with air rockfall behaviour in three different situations: free fall, bouncing on a horizontal plane and bouncing on an inclined plane.

1. INTRODUCTION

The kinematics of falling rocks has been studied analytically and experimentally for several decades (Ritchie, 1963; Pfeiffer and Bowen, 1989; Azzoni *et al.*, 1995). Almost all of these efforts have been done on land, but rockfalls also happen in the underwater environment (Edmunds *et al.*, 2006). In order to protect submarine installations such as pipelines, or to be able to protect the environment, for example coral reef, underwater rockfall analysis must be undertaken to establish block trajectory. Aside from Turmel and Locat (2007), where kinematics of underwater rockfall have been described, the only study on submarine rockfalls is from Beranger *et al.* (1998) where they introduced two constants to account for water effects (drag and added mass). In this study, underwater rockfall kinematics will be briefly described in order to introduce a new numerical model developed to model underwater rockfall. Results of simulations are presented. More details can be found in Turmel (2008).

2. UNDERWATER ROCKFALL KINEMATICS

The main difference between underwater rockfall and "dry" rockfall is the presence of water in the first one. Since the values of water density and viscosity are higher than those of air, hydrodynamic forces cannot be neglected (Turmel and Locat 2007). Four hydrodynamic forces will play an

important role in rockfall kinematics: buoyancy force, lift and drag forces and added mass force. Other hydrodynamic forces, such as Basset history force (force due to the non-stationarity of the flow) can be neglected because of the large particle diameter and the density of the rock mass (Thomas, 1992). A more detailed description of these hydrodynamic forces is made in Turmel and Locat, 2007.

2.1 Hydrodynamic forces

2.1.1 Buoyancy force

Buoyancy force is the force that obeys Archimedes principle, which states that a body immersed in a fluid is buoyed up by a force equivalent to the mass of displaced fluid (eq. 1). This force is directed upward and defined as:

$$F_b = -\rho_w Vg \quad [1]$$

Where ρ_w is water density, (V) is the volume of the displaced fluid and (g) the gravitational acceleration.

2.1.2 Lift and drag forces

When a body is moving through a fluid, an interaction between them occurs. This interaction can be described in term of shear stress (τ) due to viscous effects and in term of normal stress due to pressure (p) (Munson *et al.*, 1998). The difference between the two forces is the direction in which they are applied. Lift force (L) (eq. 2) is applied perpendicular to the direction of motion and drag force (D) (eq. 3) is applied in the direction of motion. (A) is the area of the moving body.

$$L = dF_x = p \sin \theta dA + \tau \cos \theta dA \quad [2]$$

$$D = dF_z = p \cos \theta dA + \tau \sin \theta dA \quad [3]$$

In order to calculate these forces, we need to know, at each time step, the pressure and speed gradient around the moving body.

2.1.3 Added mass force

When a body accelerates in a fluid, it accelerates a part of the fluid, and this requires energy (Brennen, 1982). The general form of this force (F_{am}) is the following (eq. 4, modified from Gondret *et al.*, 2002), where (C_a) is the added mass coefficient, depending of the density ratio between the fluid and the solid (Odar and Hamilton, 1964), V is the volume of the moving body and U the speed vector of the moving body ($U = (u_x, u_y, u_z)$).

$$F_{am} = VC_a \rho_f \frac{dU}{dt} \quad [4]$$

2.2 Bouncing motion

Laboratory experiments from McLaughlin (1968), Gondret *et al.* (2001) and Joseph *et al.* (2001, 2004) show that when Stokes number (defined as the ratio between viscous and inertial force of an object in a fluid, eq. 5) is high (> 1000), the effect of the fluid on normal restitution coefficient is negligible. In their laboratory experiments, they used spheres. Nothing has been done in the case of cubes. Because Stokes number will be over 1000, we assume the wet coefficient of restitution is the same as the dry one.

$$St = \frac{1}{9} Re \frac{\rho_s}{\rho_f} \quad \text{where} \quad Re = \frac{\rho_f U l}{\mu} \quad [5]$$

Using these assumptions, it is possible to use Colorado Rockfall Simulation Program (CRSP) formulation (Pfeiffer and Bowen, 1989) for bouncing in our model without any modifications.

3. NUMERICAL MODEL

A numerical model was chosen over the algebraic model used by Beranger *et al.* (1998). This was done to take into account all hydrodynamics forces and consider the behaviour of the fluid around the moving block. A 3D finite volume model has been constructed. The software used to develop this numerical model is OpenFOAM (Open Field Operation and Manipulation), which is a set of C++ modules used to build solvers to simulate specific problems in engineering mechanics (Weller *et al.*, 1998). The main advantage of OpenFOAM is that this software is open source, so everything in the software can be optimized and modified for the problem.

The solver used to solve the dynamics of the moving block in a fluid is based on two pre-existing solvers, which take into account turbulent flow and dynamic meshing. Because of the high Reynolds number (> 10000) (eq. 5) inherent to this problem, turbulence will be taken into account with a $k-\epsilon$ model. Dynamic meshing is the key of this solver. Since the block will move in the domain, topological modifiers will be used to model the movement. This way, the mesh structure will not be too much distorted.

This solver uses an implicit pressure-velocity iterative framework over a 3D unstructured mesh of polyhedral cells (Jasak, 1996).

3.1 Governing equations

Underwater rockfalls is a multi-physics problem involving a moving rigid body in a Newtonian fluid. In the free fall behaviour, two kinds of forces are acting on the moving body, gravity and hydrodynamic forces. The latter forces can be neglected at the time step where there is a collision. It is almost impossible to calculate hydrodynamic forces without numerical analysis because of the non-linearity of the equations. Two equations will be solved: momentum (eq. 6) and continuity equations (eq. 7).

$$\rho_f \left(\frac{\partial u_i}{\partial t} + u \cdot \nabla u_i \right) = \rho_f g + \nabla \cdot \sigma \quad [6]$$

$$\nabla \cdot u = 0 \quad [7]$$

$$\sigma = -pI + \mu_f \left[\nabla u_i + (\nabla u_i)^T \right] \quad [8]$$

Where σ is the stress tensor for a Newtonian fluid, I is the identity matrix and μ the fluid viscosity. When these equations are solved, it is possible to determine forces on the block because all pressure and velocity terms are known.

For the moving rock itself, the second law of Newton will be applied:

$$\sum F = M \frac{\partial U}{\partial t} = M \bar{g} + \bar{F}_h + \bar{F}_c \quad [9]$$

Where F_h are hydrodynamics forces and F_c are contact forces. Lift, drag and turbulent forces can be calculated numerically. When momentum and continuity equations are solved, there is a solution for the pressure and for the flow speed on the domain. By solving these equations, hydrodynamic forces can then be calculated.

3.2 Force calculation

In lift and drag forces formulas (eq. 2 and 3), one can see that these forces are due to pressure and shear stress. In this model, it is easier to formulate the equations for pressure forces and shear stress forces than using usual lift and drag formulas. A third force will be added to the model: turbulent forces. It is usual in computational fluid dynamics (CFD) to calculate turbulence properties of the flow with a turbulence model. This kind of turbulence model helps to reduce the mesh size and the time step. The last hydrodynamic force considered by this model, buoyancy force, is held constant in the problem because we impose incompressibility of the fluid. The Pressure in the problem is a relative pressure and not an absolute pressure, consequently, momentum equations have been modified for this particularity.

When these forces have been calculated, the model computes the summation of forces on the domain, taking into account gravity force. Acceleration of the block can be calculated and a displacement can be imposed on the block.

3.2.1 Pressure force

When knowing the pressure around the moving block, it is possible to sum these pressures. The resultant will be a vector because we are solving for a 3D mesh. Forces are calculated around the block by multiplying this vector by the water density.

$$\text{Pressure} = \sum p * \text{mesh.Sf} () \quad [10]$$

$$\text{PressureForce} = \text{Pressure} * \text{medium.Rho} () \quad [11]$$

Where p is the pressure on one face area of the block, $\text{mesh.Sf}()$ is the face area vector of this face and $\text{mediumRho.value}()$ is the water density. The summation in eq. 10 is done over all face areas of the block. (These formulas use OpenFOAM code formulation).

3.2.2 Viscous forces

To calculate viscous forces around the block, we have to know the flow speed and the gradient of this speed. As for the pressure forces, a summation over all the face areas of the block will be done on the gradient of the speed. Then, this will be multiplied by the viscosity and the water density to calculate the forces.

$$\text{Viscous} = \sum \text{U.grad} () * \text{mesh.magSf} () \quad [12]$$

$$\begin{aligned} \text{ViscousForce} = & \text{Viscous} * \text{medium.Rho} () \\ & * \text{nuMean.value} () \end{aligned} \quad [13]$$

Where U.grad is the gradient of the speed on the area face, mesh.magSf is the face area magnitude and nuMean.value is the viscosity of the fluid.

3.2.3 Turbulent forces

When solving for a turbulent flow, the fluid velocity is separated in an average fluid velocity and a fluctuating fluid viscosity due to turbulence. The Reynolds Stress Tensor (R) is the stress tensor in a fluid due to the random turbulent fluctuations in fluid momentum. This stress is obtained from an average of these fluctuations.

As for the two last forces (pressure and viscous forces), the force due to turbulence will be a summation, over the block, of Reynolds stress tensor multiplied by water density.

$$\text{Turbulence} = \sum \text{R} * \text{mesh.Sf} () \quad [14]$$

$$\text{TurbulenceForce} = \text{Turbulence} * \text{medium.Rho} () \quad [15]$$

3.3 Dynamic meshing

A topological change in a mesh is a change in mesh connectivity: *i.e.*, adding or removing layers of volumes without complete re-meshing. The mesh modifications in our case involved layer addition/removal. The goal of these is to create a new layer of volumes if the gap between the modifiers and the adjacent layers is too large or to remove a layer if this gap becomes too small (figure 1) (Jasak, 2006). In our case, two mesh modifiers have to be added for free-fall motion and rebound on a horizontal plane, while four of them are necessary for rebound on an inclined plane. Having two mesh modifiers allows the block to be moving in 1D, while four allows the block to be moving in 2D.

3.4 Numerical procedure

Modeling rockfalls in underwater environment requires many steps in the model. At time 0, pressure and flux speed are set and boundary conditions are imposed. Mesh modifiers are implemented at this stage. Then, at each time step, forces around the block and acceleration of the block are calculated and a displacement is imposed to the mesh section between mesh modifiers. Mesh modifiers will check if there is need for topological changes, if so, it is done. At this point, momentum (eq. 6) and continuity (eq. 7) equations are solved and correction for turbulence is done. Results will be saved. Solver will then proceed to the next time step. Momentum and continuity equations are solved using a PISO modified loop (Jasak, 1996).

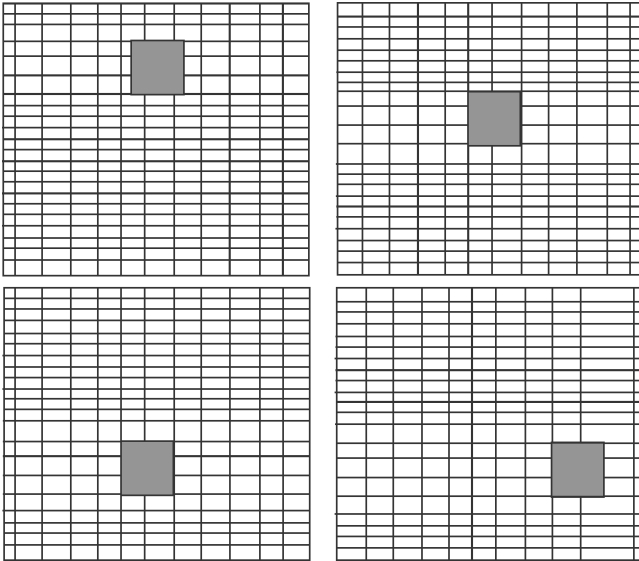


Figure 1 Example of topological change. The first three images show a vertical displacement and the last one shows a horizontal displacement. The grey square represents the block.

4. RESULTS OF SIMULATION AND DISCUSSION

The rockfall software has been developed to evaluate both trajectory and impact forces. In our case, we limit ourselves to the three basic phases of this phenomenon: free fall, bouncing on a flat plane and bouncing on an inclined plane. Our goal is to compare these different phases in water and in air. To do so, simulations done with the present model have been compared with simulations done with RocFall software (from RocScience). For the latter simulations, aerodynamics effects are neglected. If we negate hydrodynamic forces in our simulation with the model developed, we do obtain the same results as with RocFall.

A large number of simulations have been run to look at the effect of mesh grading and time-step. Results presented here are for simulations where increasing the mesh size or decreasing time-step does not change solutions. The parametric study with these parameters is presented in Turmel (2008).

4.1 Free fall

Free fall simulations will be used to validate the numerical model with experimental results. Then, effects of three different factors (block density, block size and water current) will be presented.

4.1.1 Model validation with experimental results

Numerical results for the 0.069m^3 block with a density of 2500 kg/m^3 , have been compared with experimental results obtained on the Coriolis II research vessel ship, where a $0.3\times 0.3\times 0.3\text{m}$ concrete block has been released in the St-Lawrence River (Quebec) (figure 2). The block was attached

to the boat with a rope, on which colour marks were drawn at each metre. A movie of the rope during the free fall was recorded, and the final speed of the block was measured with a frame-by-frame analysis of the movie (figure 3). The block was released three times, but due to experimental difficulties in the first one, only two launches have been analysed. The terminal velocity of the block has been calculated to be 2.2m/s (± 0.2).

Figure 4 shows results for all free fall numerical simulations. On this diagram, speed is negative because positive z axis is directed upward. The terminal speed for the 0.069 m^3 block with a density of 2500 kg/m^3 is of 1.7 m/s , which is in accordance with results obtained experimentally. The difference between the velocity found in the experiments and in the numerical model may be explained by many factors, such as the fact that the block in the water may have rotated (smaller drag coefficient), or that boat speed and water current were not equal to zero.

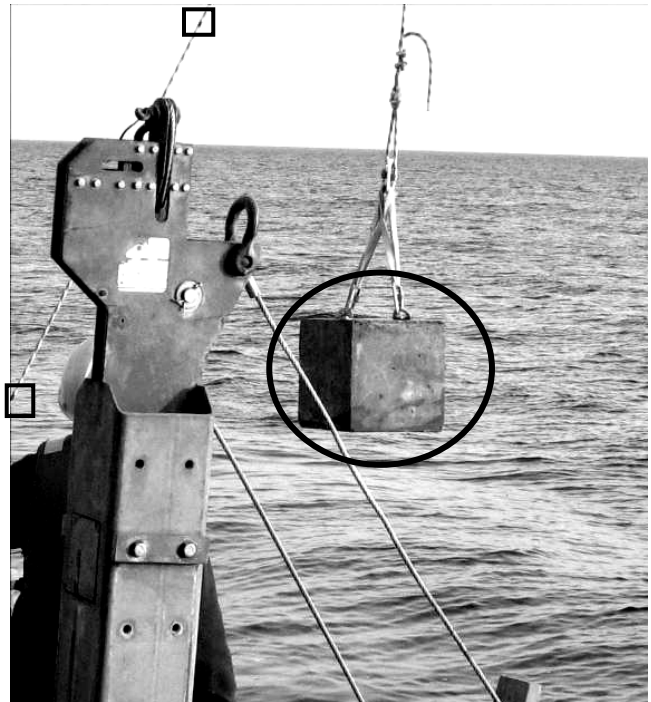


Figure 2 Experimental setting. Black circle indicates the concrete block and black rectangles indicates marks on the rope.

4.1.2 Effect of block density

Block density has been changed for the smallest block, ranging from 2400 to 2700 kg/m^3 . It can be seen on figure 4 that rock density plays an important role in rockfall behaviour, the higher the density, the higher the terminal speed will be. These blocks will reach their terminal speed within less than 2 meters of free fall. This effect is due to

acceleration that increases with an increase of rock density (eq. 9). Surface of contact between the block and water stays the same, as for the buoyancy force. This is very different from usual rockfall simulations, where block is treated as a particle in free fall motion (Stevens, 1998) and mass is used only for energy calculations.

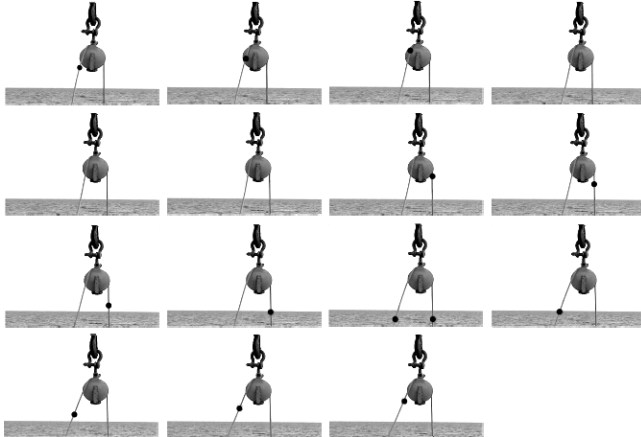


Figure 3 Frame by frame analysis, each frame are separated of 1/30 sec. Black dot on the rope represents marks drawn at each meters on the rope.

4.1.3 Effect of block size

Block size is a very important factor to take into account when doing underwater rockfall simulations. Increasing block size has an effect on the mass of the block and on the surface of contact between the block and the water. The frontal area will be greater, but mass will be higher. These effects work antagonistically, the first one will increase the speed of the block and the second one will decrease it.

After 1.2 seconds of free fall, there is a difference of about 0.8m/s (vertical speed) between the 0.3m length block and the 0.5m length. Even for the 1m³ block, the difference of speed after 1 second of free fall in water and in air is considerable. Vertical speed under water will be approximately 3m/s, while in air it will be 9.8m/s.

4.1.4 Effect of water current

Figure 5 shows the effect of the water current when a 1m/s current is applied on the 0.069m³ block. Water current will affect the block movement because it introduces horizontal forces on it. When simulation time is very large, horizontal speed of the block will be the same as water current.

4.2 Bouncing on a horizontal plane

Bouncing on a horizontal plane is a particular case of bouncing because it involves only a movement in one axis. This scenario has been modeled for two block sizes: 0.069m³ and 1m³. Figure 6 and 7 shows displacement of these blocks versus time and force due to pressure versus time. For the first one, speed at contact is the terminal

speed of the block. For the second one, speed at contact is the speed after 6.2 meters of free fall. The third graphic shows comparison between Beranger *et al.* (1998) model and this present model for the 0.069m³ block with a density of 2500 kg/m³. For all these simulations, R_n and R_t have been set to 0.8.

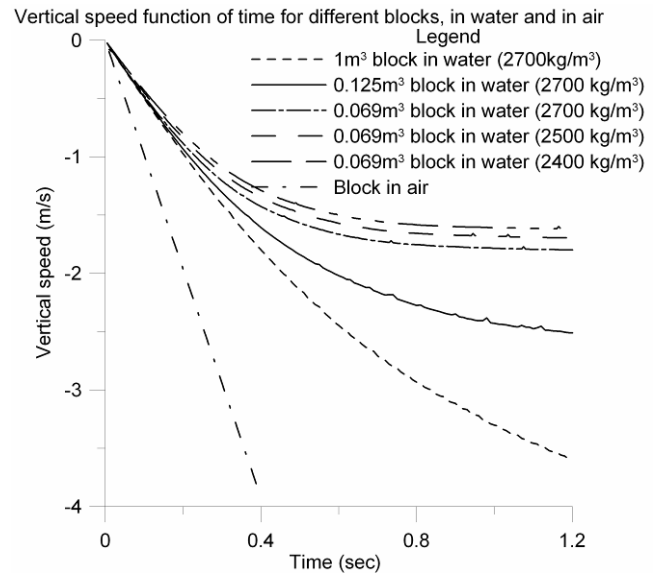


Figure 4 Vertical speed function of time for different blocks

In our model, the contact is not on the wall but at about 10 centimetres from the wall. This is due to an artifact of the dynamic meshing algorithm that needs a minimum of two layers of cells to be working.

Both cases show a similar behaviour. Rebound after contact is very small, compared to what can be seen in air. Pressure force around the block, which is almost constant before the rebound, shows a very high drop right after contact. This is due to the mass of water that has been accelerated by the moving block before contact. When block rebounds on the surface, the mass of water is acting against the movement. Decelerations of more than 20m/s² are seen just after contact. Decelerations of more than gravitational acceleration are acting on the block during all the upward movement. With such high pressure force acting on a small block like this one, it can be expected that forces applied on the wall by the block are quite high. These forces have not been calculated in this work.

Comparison between this model and the Beranger *et al.*, (1998) model can be made for such a case. In their model, they have added a constant drag coefficient (set at 2.0 here, for the case of a rectangle), and a constant added mass coefficient (0.5) in a modified formula used by Pfeiffer *et al.* (1989). An exact equation can be formulated from these differential equations and these equations have been applied for the rebound on a horizontal surface (figure 8). Coefficients of restitution are also set at 0.8.

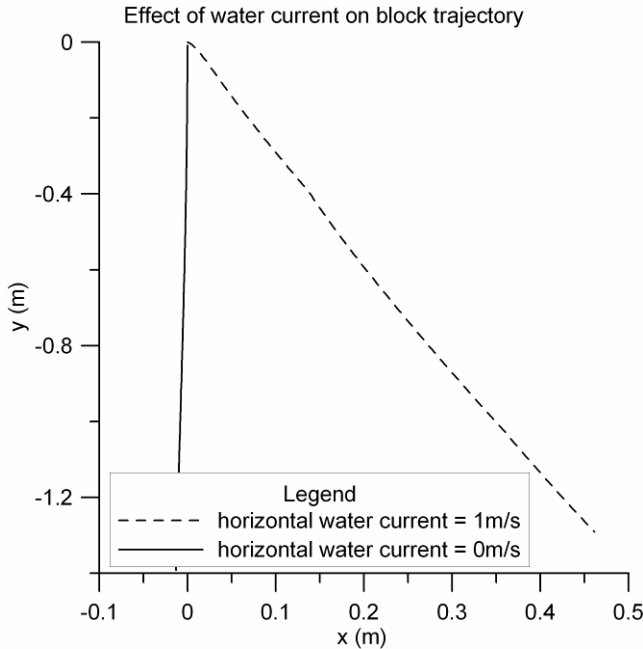


Figure 5 Effect of the water current on block trajectory

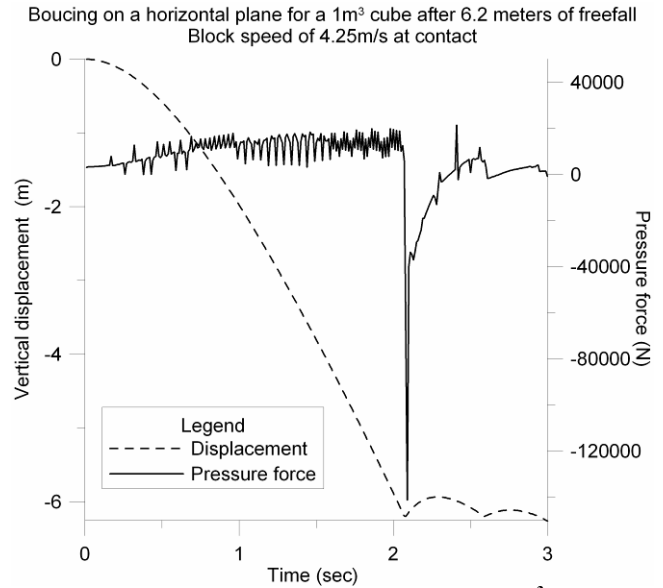


Figure 7 Bouncing on a horizontal plane for the 1m³ block

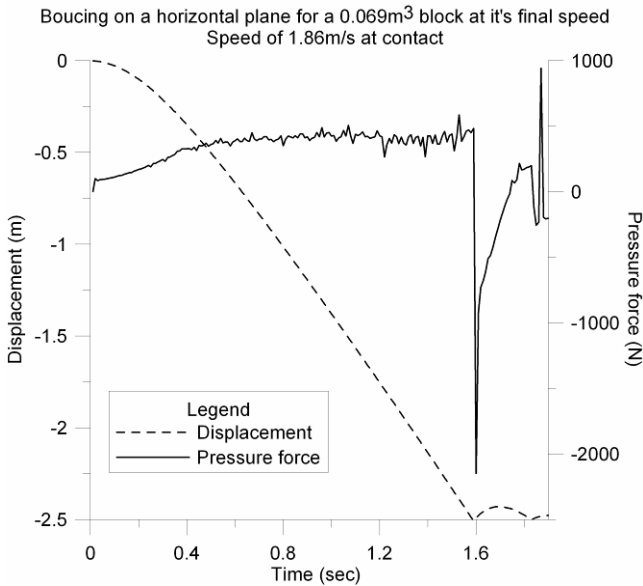


Figure 6 Bouncing on a horizontal plane for the 0.069m³ block

It can be seen that the Beranger *et al.* (1998) simplified model predicts quite well free fall behaviour, but because they do not take into account the fluid movement behind the block, their model cannot predict bouncing motion. Rebound height calculated by Beranger *et al.* model is more than 5 times higher than with the present model.

It should be stated that if the rebound occurred at the wall, an overpressure will certainly occur between the block and the wall, caused by the flow of fluid trapped between them. This will probably cause a deceleration of the block before the contact, so the speed after contact will be smaller than the modeled one.

4.3 Bouncing on an inclined plane

Two different plane angles have been imposed for inclined plane: 45 degrees and 30 degrees. Figures 9 and 10 illustrate a comparison of the block trajectory between Rockfall software simulation and this model simulation. Vertical speed at contact is set the same in air and in water, at 1.86 m/s, for both cases. This way, it is possible to observe effect of hydrodynamic forces on the impact after rebound and not only the effect of the difference of speed at contact.

For these simulations, even if it may have a significant impact on the results, rotation has been ignored in water due to limitations with dynamic meshing algorithm. Normal and tangential coefficients of restitution were set at 0.8 and surface roughness has been set at 0. This model has not been compared to the Beranger *et al.* (1998) model because the way they calculate forces in a scenario like this one was not clearly formulated.

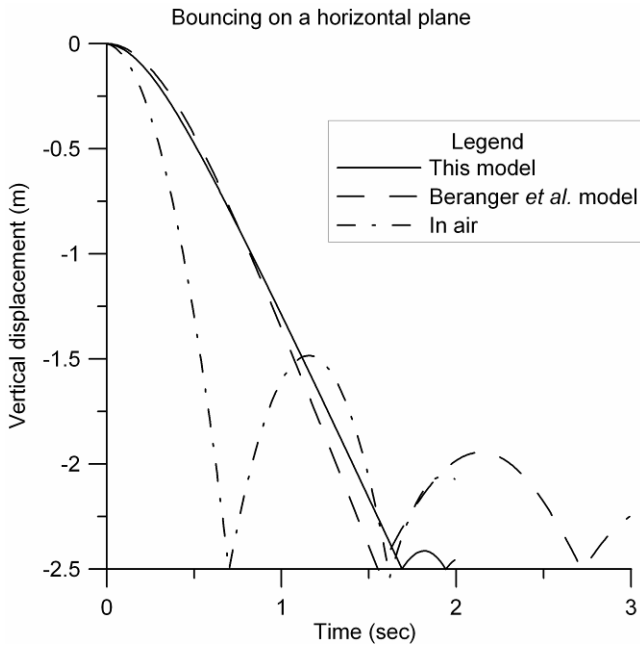


Figure 8 Comparison between this model, Beranger *et al.* model and air behaviour (calculated by the model presented here)

As can be expected, travel distance of the block is lower in water than in air. In figure 9, the slope was inclined at 45 degrees. After contact, vertical speed is transformed into horizontal speed because the movement is purely vertical before contact. The effect of the water moving vertically will be attenuated by this change of movement direction and block is accelerating in vertical direction quite slowly (because of hydrodynamic forces still acting on it). In addition, these forces are also acting in horizontal direction, and there are no gravitational accelerations in this direction. Horizontal block speed will decrease because of hydrodynamic forces. In this first case, the block will reach an horizontal distance of less than 2/3 of the distance reached by the same block in air before next rebound.

The same result is obtained when there is a rebound on a 30 degrees plane (figure 10). Additionally, like in the horizontal bouncing case, vertical speed will decrease very fast, compared to the air case. Horizontal decelerations will also be greater because when the block is trying to move in a horizontal direction, in a flow going in a vertical direction, it takes a lot of energy to change flow direction. For this scenario, block under water will reach a horizontal distance less than half of the distance reached by the same block in air.

With these results, it can be expected that the higher is the inclination of the plane, the smaller the difference between the block trajectory in air and water will be, and *vice-versa*. The main difference will be the speed of the block for the subsequent impacts and the energy of this block. If the speed is smaller, the energy of the block is smaller.

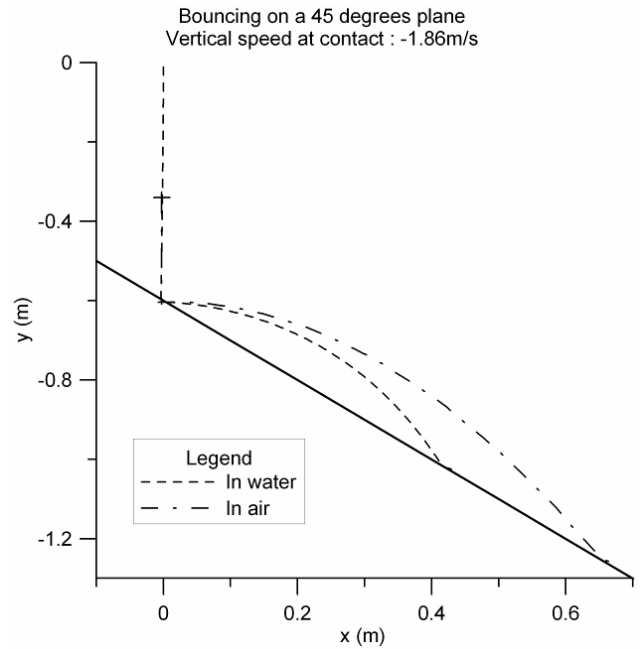


Figure 9 Bouncing on a 45 degrees plane

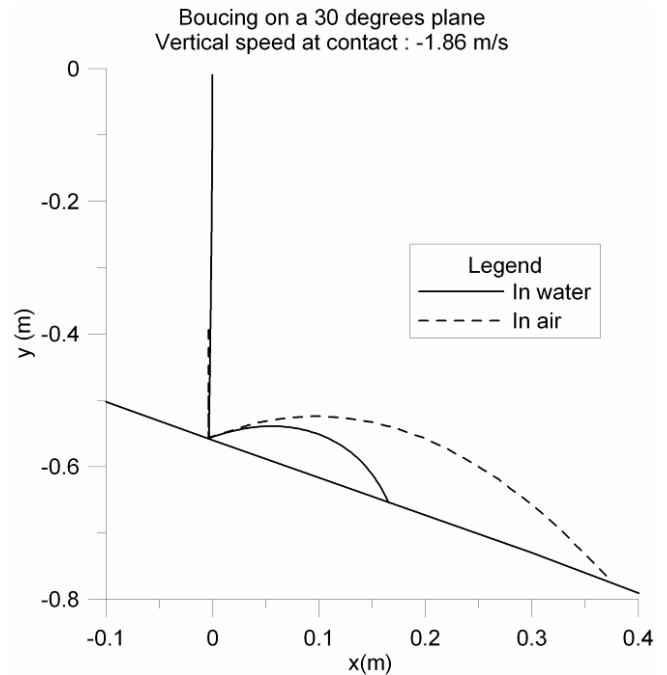


Figure 10 Bouncing on a 30 degrees plane

5. CONCLUSION

A numerical model has been developed to compute forces around a moving block when subjected to hydrodynamic forces. A comparison was carried out for underwater rockfall behaviour with air rockfall behaviour in three different situations: free fall, bouncing on a horizontal plane and on an inclined plane, and the effect of water current in

free fall. Hydrodynamic forces do play a very important role on these behaviours.

The terminal speed of the falling block will be slower in water than in air. In the free fall case, block behaviour will be influenced by its density and size. The bigger the block is, or the denser it is, the greater the terminal speed will be. Water current will also modify rockfall trajectory.

When the block is bouncing on a horizontal or on a gentle slope, the mass of water accelerated by the block in the free fall will push against the block and contribute to decelerate it. Pressure force due to the water will also decrease the horizontal speed. This force will limit the horizontal trajectory of the block. When the slope is steeper, the effect of the mass of water is not so important. Horizontal forces, decelerating the horizontal speed of the block, will also reduce horizontal displacement of the block.

6. ACKNOWLEDGEMENTS

The authors would like to thank both reviewers, O. Hungr and J. Hadjigeorgiou for their constructive remarks. We also thank the Natural Sciences and Engineering Research Council of Canada for their financial support.

7. REFERENCES

- Azzoni, A., La Barbera, G., Zaninetti, A., (1995). *Analysis and Prediction of Rockfalls Using a Mathematical Model*. International Journal of Rock Mechanics and Mining Sciences and Geomechanical Abstracts 32(7) pp.709-724
- Beranger, M., Carpaneto, R., Boote, D., & Zuccarelli, F. (1998). *Probabilistic assessment of rockfall impact on subsea pipelines*. OMAE 1998: Proceedings of the 17th international conference on offshore mechanics and arctic engineering. Portugal.
- Brennen, C. E. (1982). *A review of added mass and fluid inertial forces*. California: Naval Civil Engineering Laboratory, Port Hueneme.
- Edmunds, M., Bryant, C., Crozier, J. Gilmour, P., Pickett, P., Stewart, K., & Williams, J. (2006). *Port Phillip Bay Channel Deepening Project. Trial Dredging Experiment: Deep Reef Impact Assessment. Report to Port of Melbourne Corporation*. Australian Marine Ecology Report 334, Melbourne.
- Gondret, P., Lance, M., & Petit, L. (2002). *Bouncing motion of spherical particles in fluids*. Physics of fluids , 14 (2), pp. 643-652.
- Jasak, H., (2006), *Multi-physics simulations in continuum mechanics*, 5th International Congress of Croatian Society of Mechanics
- Jasak, H., (1996), *Error analysis and estimations for the finite volume method with applications to fluid flows*, Ph.D. Thesis, Imperial College, University of London.
- Joseph, G., & Hunt, M. (2004). *Oblique particle-wall collisions in a liquid*. Journal of Fluid Mechanics , 510, pp. 71-93.
- Joseph, G., Zenit, R., Hunt, M., & Rosenwinkel, A. (2001). *Particle-wall collisions in a viscous fluid*. Journal of Fluid Mechanics , 433, pp. 329-346.
- McLaughlin, M. (1968). *An experimental study of particle-wall collision relating of flow of solid particles in a fluid*. Engineer's degree thesis, California Institute of Technology, Pasadena, California
- Munson, B. R., Young, D. F., & Okiishi, T. H. (1998). *Fundamentals of Fluid Mechanics (éd. 3rd edition)*. New-York: John Weiley & Sons, Inc.
- Odar, F., & Hamilton, W.S. (1964). *Forces on a sphere accelerating in a viscous fluid*. Journal of fluid mechanics (18), pp. 302-314
- Pfeiffer, T., & Bowen, T. (1989). *Computer Simulation of Rockfalls*. Bulletin of the Association of Engineering Geologists , XXVI (1), pp. 135-146.
- Ritchie, A.M., (1963). *Evaluation of rockfall and its control*. Highway Research Record (17), pp. 13-28.
- Stevens, WD., (1998). *Rockfall : a tool for probabilistic analysis, design of remedial measures and prediction of rockfall*, M. Sc Thesis, University of Toronto
- Thomas, P.J.. (1992). *On the influence of the Basset history force on the motion of a particle through a fluid*. Brief communications, Phys. fluids A, 4 (9), pp. 2090-2093
- Turmel, D., Locat J. (2007). *Underwater Rockfall Kinematics: A preliminary analysis*, Submarine Mass Movements and Their Consequences; pp. 139-147
- Turmel, D. (2008), *Analyse sub-aquatique des chutes de blocs*, M.Sc. Thesis, Université Laval
- Weller, H., Tabor, G., Jasak, H., Fureby, C., (1998), *A tensorial approach to computational continuum mechanics using object-oriented techniques*, Computers in physics, 12 (6), pp. 620-631